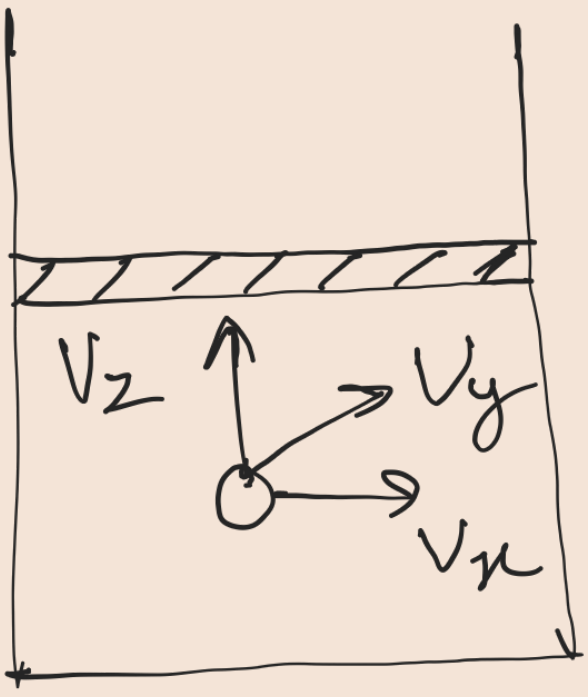
Initial Condition



Let 'V_z' be the velocity direction of gas molecules towards piston. Under initial state, $\left(V_x^2\right)_{avg} = \left(V_y^2\right)_{avg}$

$= \left(V_z^2\right)_{avg} = \dfrac{\left(V^2\right)_{avg}}{3}$.

Also, the internal energy initial U_i can be expressed as

$\frac{1}{2}(nM)(V^2)_{avg}$. n = no. of moles & M = molar mass.

Now two important obs'.

(i) → $V_{rms} >> V_{piston}$.

So wrt gas molecules, piston movement can be assumed almost quasi-static.

(ii) mean collision time is excessively high, which means $(V_x^2)_{avg}$ & $(V_y^2)_{avg}$ remain unaffected during piston movement.

Due to quasi-static piston movement, the final pressure becomes half due to double volume.

In final state,

$(V_x^2)_{avg}$ & $(V_y^2)_{avg}$

remain unaffected. But $\langle V_z \rangle$ for single molecule gets halved, due to which $(V_z^2)_{avg\,(new)}$ is $\frac{1}{4}(V_z^2)_{avg\,(old)}$.

$$\left(V_x^2\right)_{\substack{avg \\ new}} = \left(V_y^2\right)_{\substack{avg \\ new}} = \frac{\left(v^2\right)_{avg}}{3}$$

But $$\left(V_z^2\right)_{\substack{avg \\ new}} = \frac{1}{4}\left(V_z^2\right)_{\substack{avg \\ old}}$$

$$= \frac{\left(V^2\right)_{avg}}{12}.$$ Hence,

$$U_{new} = \frac{1}{2}(nM)\left(\left(V_x^2\right)_{avg} + \left(V_y^2\right)_{avg} + \left(V_z^2\right)_{avg}\right)$$

$$= \frac{1}{2}(nM)\left[\frac{1}{3} + \frac{1}{3} + \frac{1}{12}\right]\left(V^2\right)_{\substack{avg \\ new}}$$

$$= \boxed{\frac{3}{4}}\left(\frac{1}{2}(nM)\left(V^2\right)_{avg}\right)_{old}.$$